A note on twin primes and a natural generalisation – Peter Braun

The Twin Prime Problem (and generalisation)

Theorem

Let a(1), a(2) ...a(N) be any sequence of counting numbers with

a(1) < a(2) < a(3) < ...< a(N)

and the property that there is no full residue class modulo any prime number in

{a(1), a(2), …a(N)}.

Now let

P (N) ≡ There exist unbounded numbers of n such that

n+a(1), n+a(2), ..., n+a(N) are all prime numbers.

The generalisation is that P(N) is true for N≥ 1 for any choice of sequence.

P(1) is the theorem about an unbounded number of prime numbers and P(2) includes the twin prime problem.

Lemma:

Let A be a set of natural numbers A={a(1), a(2), … a(N)} with a(1)< a(2) < …< a(N) and such that A does not contain a complete residue set modulo any prime.

There exist unbounded natural numbers n such that

n+a(1), n+a(2), …., n+a(N) are mutually coprime and each is coprime to any nominated product of prime numbers.

Proof:

Let A+m denote the set {m+a(1), m+a(2),…, m+a(N)}

The only primes which are possibly divisors of more than one number in A+m are prime divisors of Π(a(i)-a(j)) with i ≠ j.

If there is such a prime denote any choice by p.

If there is no such p then select an arbitrary prime p.

Now let r(p) be the missing residue modulo p in A

At least one of the sets

A+1, A+2 ,…A+p is missing the residue 0 modulo p

Indeed, each set has a missing residue modulo p and it is a different residue for each set. Let A+r(p) contain only elements coprime to p.

Now let q be any other prime divisor of $\Pi(a(i)-a(j))$ with $i \neq j$.

If no such q exists let q be any prime other than p.

At least one of the sets

A+r(p) + 1.p, A+r(p) + 2.p , ……., A+r(p) + q.p

has the property that each element is coprime to pq

Indeed, each set has a missing residue modulo q and they are different in each case.

One of the sets is missing the residue o modulo q.

The lemma follows as a simple extension of this argument.

Corollary: There exist an unbounded number of prime numbers

Proof:

We have the existence of a set of numbers coprime to p(1)p(2) …p(n).

We see from the preceding lemma that the ''conditions' for the assertion are present but it remains to argue that the conditions are 'sufficient'.

Sketch of argument for main theorem

This is discussed under the assumption that P(τ) false is rejected and relies on accepting the reasoning in the preceding note.

For each number N

P(N) is a proposition about N

$P(N) \rightarrow P(N-1)$ (true)

P(1) is true

$P(\tau)$ is unprovable.

Then $P(N)$ may be taken to be true for all N without contradiction.

Proof (program style)

Let $N=2$

Step 1

Now suppose $P(N)$ is false. ($P(N) \rightarrow P(N-1)$ is okay)

Then $P(N+1)$, $P(N+2)$, … $P(\tau)$ are false

But $P(\tau)$ is unprovable.

Hence $P(N)$ is not false

Substitute $N+1$ for N in step1

Outcome $P(2)$, $P(3)$ …. are not false

Step 2

Now suppose $P(N)$ is true. ($P(N) \rightarrow P(N-1)$ is okay)

Then $P(N+1)$ is either true or false

If $P(N+1)$ is false go to step1

Otherwise $P(N+1)$ is true

$N=N+1$ in step2

Repeat as necessary

Hence $P(1)$, $P(2)$, $P(3)$ … may be taken to be true avoiding contradiction.

It would be interesting to discover what issues arise in an attempt to construct a counter example.

For the moment the argument form is accepted.

From the discussion we deduce that instances of K , $K+2$ which are both prime will continue to be discovered in numerical calculations since $\{0,2\}$ does not contain a full residue class modulo any prime.

The reader with an elementary understanding of arithmetic should note that τ is a construct (axiom) and there is no point in thinking about its existence in any metaphysical sense.

A similar axiom which is aided by a picture or image is found in projective geometry where parallel lines meet at 'infinity'.

The area in number theory called sieve theory develops methods which may be applied to these sorts of problems.

The power of these methods is remarkable given the difficulties involved.

However, whether sieve methods will be able to converge on the general questions seems an open question.

References

The interested reader is referred to Introduction to Mathematical Philosophy by Bertrand Russell , Eleventh Impression 1963, and printed by Neill@ Co. Ltd., Edinburgh. The book is in the Muirhead Library of Philosophy.

Another useful reference is the student text Theory and Problems of Modern Algebra by Frank Ayres, Jr. in the popular Schaum's Outline Series published by the McGraw-Hill Company June 1965.

The interested reader is also referred to the last chapter of Sieve Methods by H. Halberstam and H. –E Richert published by Academic Press (London) in 1974.

The preceding link to the pdf 'A note on the Riemann hypothesis by P Braun' gives background on why it is appealing to include τ in the arithmetic axioms.

Website: www.peterbraun.com.au